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MAGNETOHYDRODYNAMIC HEAT TRANSFER IN A CIRCULAR DUCT FOR TEMPERATURE BOUNDARY CONDITION OF THE THIRD KIND

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Abstract—To investigate the influence of the temperature boundary condition of the third kind on the magnetohydrodynamic heat transfer in the thermal entrance region of a circular duct, the energy equation is solved by applying the Galerkin–Kantorowich method of variational calculus. The fully developed velocity profile is assumed and the heat generation within the fluid is neglected. It is concluded that there can be a significant influence of the Biot number on the local Nusselt number. Representative results are depicted in tables.

NOMENCLATURE

- A, channel cross section;
- B, magnetic induction;
- Bi, Biot number, equation (4);
- D, a matrix, equation (9);
- F, a vector, equation (6);
- Ha, Hartmann number, equation (4);
- N, number of terms in a series;
- Nu, Nusselt number, equation (15);
- Pe, Peclet number, equation (4);
- Pr, Prandtl number, equation (4);
- R, a vector, equation (11);
- Re, Reynolds number, equation (4);
- T, temperature;
- W, a matrix, equation (9);
- c, tube radius, Fig. 1;
- c_p, specific heat at constant pressure;
- d_h, hydraulic diameter;
- f, a function, equation (6);
- g, a function, equations (19), (20);
- h, heat-transfer coefficient;
- k, overall heat-transfer coefficient;
- p, pressure;
- q, heat flux;
- r, radial coordinate;
- s, characteristic value;
- v, velocity;
- x, axial coordinate;
- [ ], matrix;
- { }, column vector.

Greek symbols

- β, eigenvalue, equation (18);
- η, dynamic viscosity;
- λ, thermal conductivity;
- μ, magnetic permeability;
- ν, kinematic viscosity;
- ρ, mass density;
- σ, electrical conductivity.

Subscripts

- a, ambient;
- i,j, running index;
- m, mean value;
- r, radial coordinate direction;
- w, wall;
- x, axial coordinate direction;
- 0, prescribed value.

Superscripts

- $\bar{}$, dimensionless quantity, equation (4);
- $'$, $(d/d\bar{x})$, equation (9).

INTRODUCTION

IN THE previous analysis on the magnetohydrodynamic (MHD) laminar forced convection heat transfer in the thermal entrance region of a circular duct, the boundary condition of the second kind characterized by the prescribed wall heat flux is assumed [1]. A more realistic condition in many applications, however, will be the temperature boundary condition of the third kind: the local wall heat flux is a linear function of the local wall temperature. This situation is encountered in the heat-transfer process, where the radiative heat transfer, describable in terms of Newton's law of cooling, occurs at the channel wall and is, to the author's knowledge for the MHD flow in a circular duct, not reported in the literature.

The objective of the present paper is to investigate the MHD laminar forced convection heat transfer in the thermal entrance region of a circular duct for the temperature boundary condition of the third kind. Assuming constant fluid properties and fully developed Hartmann flow, the energy equation is solved by employing the Galerkin–Kantorowich method of variational calculus. Since the main concern of this analysis is with the influence of the finite wall thermal resistance on the heat transfer, the axial heat conduction and the heat generation within the fluid are not considered.

2. ANALYSIS

Consider steady, fully developed laminar MHD flow in a circular duct (Fig. 1). For the region $x \geqslant 0$, where a constant ambient temperature is maintained, a uniform magnetic field is imposed perpendicular to flow direction and no external electric field is applied.

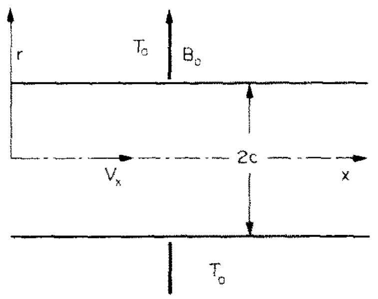

FIG. 1. Circular duct under investigation.

The flow considered is similar to the Hartmann flow in a flat channel, the Hall effect and induced magnetic fields are omitted. Assuming constant fluid properties and neglecting the heat conduction in flow direction and the heat generation within the fluid, the laminar forced heat convection subject to boundary condition of the third kind for temperature can be described by equations [1, 2]:

$$L(\bar{v}_x) = -Re(\mathrm{d}\bar{p}/\mathrm{d}\bar{x}) + (1/\bar{r})(\mathrm{d}\bar{v}_x/\mathrm{d}\bar{r}) + (\mathrm{d}^2\bar{v}_x/\mathrm{d}\bar{r}^2) - (Ha)^2\bar{v}_x = 0, \quad (1)$$

$$L(\bar{T}) = Pe\bar{v}_x(\partial\bar{T}/\partial\bar{x}) - (1/\bar{r})(\partial\bar{T}/\partial\bar{r}) - (\partial^2\bar{T}/\partial\bar{r}^2) = 0, \quad (2)$$

$$\left.\begin{aligned} &\bar{x} = 0:\ T = T_0,\ \bar{T} = 1,\ p = p_0; \\ &\bar{x} \to \infty:\ T \to T_a,\ \bar{T} \to 0; \\ &\bar{r} = 0:\ \mathrm{d}\bar{v}_x/\mathrm{d}\bar{r} = 0,\ \partial\bar{T}/\partial\bar{r} = 0; \\ &\bar{r} = 1:\ \bar{v}_x = 0,\ (\partial T/\partial r) + k(T - T_a) = 0, \\ &\qquad (\partial\bar{T}/\partial\bar{r}) + Bi\bar{T} = 0; \end{aligned}\right\} \quad (3)$$

where k is the overall heat-transfer coefficient based on the wall thermal resistance and on the ambient side surface resistance. The dimensionless quantities employed are

$$\left.\begin{aligned} &\bar{x} = x/c,\ \bar{r} = r/c, \\ &\bar{v}_x = v_x/v_{x,m},\ v_{x,m} = (1/A)\int_A v_x\,\mathrm{d}A, \\ &\bar{p} = (p - p_0)/(\rho v_{x,m}^2), \\ &\bar{T} = (T - T_a)/(T_0 - T_a), \\ &Re = v_{x,m}c/\nu,\ Pr = \nu\rho c_p/\lambda, \\ &Ha = cB_0(\sigma/\eta)^{1/2}, \\ &Pe = RePr = \rho v_{x,m}cc_p/\lambda,\ Bi = ck/\lambda. \end{aligned}\right\} \quad (4)$$

The solution of the momentum equation (1) is

$$\left.\begin{aligned} &\bar{v}_x = Ha[I_0(Ha) - I_0(Ha\bar{r})]/ \\ &\qquad [HaI_0(Ha) - 2I_1(Ha)], \\ &-Re(\mathrm{d}\bar{p}/\mathrm{d}\bar{x}) = (Ha)^3 I_0(Ha)/ \\ &\qquad [HaI_0(Ha) - 2I_1(Ha)], \\ &\bar{v}_{x,m} = 1, \end{aligned}\right\} \quad (5)$$

where I_0 and I_1 are modified Bessel functions of order zero and one respectively.

To solve the energy equation (2), the Galerkin–Kantorowich method of variational calculus is employed, which allows to reduce a partial differential equation to an ordinary one [3]. Let the approximate temperature field be

$$\begin{aligned} &\bar{T} = \sum_j J_0(s_j\bar{r})f_j(\bar{x}), \\ &\{F\} = \{f_1(\bar{x}), f_2(\bar{x}), \ldots f_N(\bar{x})\}, \\ &J_0(s_j) - (s_j/Bi)J_1(s_j) = 0, \end{aligned} \quad (6)$$

where J_0 and J_1 are Bessel functions of first kind of order zero and one respectively. The characteristic values s_j are to be determined so that the boundary condition is satisfied. Taking the energy equation (2) with the natural boundary condition (3) as the Euler equation of the variational formulation, one may solve

$$\int_0^1 L(\bar{T})\delta\bar{T}\bar{r}\,\mathrm{d}\bar{r} = 0 \quad (7)$$

to evaluate the unknown functions $f_j(\bar{x})$. With

$$\delta\bar{T} = \sum_j (\partial\bar{T}/\partial f_j)\delta f_j \quad (8)$$

from equations (2) and (7), one can derive a system of ordinary differential equations for f_j as follows (for details see Appendix)

$$Pe[D]\{F'\} = [W]\{F\}. \quad (9)$$

To calculate the value of f_j at the channel entrance, one may use the condition

$$\int_0^1 \bar{v}_x[\bar{T}(\bar{x} = 0) - \bar{T}_0]^2\bar{r}\,\mathrm{d}\bar{r} \to \min. \quad (10)$$

From equations (6) and (10), one can deduce a system of algebraic equations for $f_j(\bar{x} = 0)$ as follows (for details see Appendix)

$$[D]\{F(\bar{x} = 0)\} = \{R\}. \quad (11)$$

The characteristic quantities describing the heat transfer at the duct wall are

$$(\partial\bar{T}/\partial\bar{r})_w = -\sum_j f_j(\bar{x})s_jJ_1(s_j), \quad (12)$$

$$\bar{T}_w = \sum_j f_j(\bar{x})J_0(s_j), \quad (13)$$

$$T_m = \int_A Tv_x\,\mathrm{d}A \Big/ \int_A v_x\,\mathrm{d}A, \quad (14)$$

$$\bar{T}_m = 2\sum_j f_jR(j), \quad (15)$$

$$Nu = hd_h/\lambda = 2c(\partial T/\partial r)_w/(T_w - T_m) = 2(\partial\bar{T}/\partial\bar{r})_w/(\bar{T}_w - \bar{T}_m). \quad (16)$$

3. RESULTS

To investigate the influence of Biot number on the MHD heat transfer in a circular duct, equations (9) and (11) were solved by employing the standard Crank–Nicolson procedure for the different values of Hartmann number.

Slug flow (Ha $\to \infty$)

For $Bi > 0$, one can derive an analytical expression for the temperature field as [4]

$$\left.\begin{aligned} \bar{T} &= 2\sum_j \frac{\exp(-s_j^2\bar{x}/Pe)J_0(s_j\bar{r})J_1(s_j)}{s_j[J_0^2(s_j)+J_1^2(s_j)]}, \\ BiJ_0(s_j) &- s_jJ_1(s_j) = 0. \end{aligned}\right\} \quad (17)$$

The special case of $Bi \to 0$, which corresponds to the case of constant wall heat flux, cannot be treated with equations (6) and (17), since the wall heat flux tends to zero. For the case of constant wall heat flux, one can deduce an analytical expression for the temperature field as [4]

$$\left.\begin{aligned} \frac{T-T_0}{cq_w/\lambda} &= 2\bar{x}/Pe+\bar{r}^2/2-1/4 \\ &\quad -2\sum_j \frac{\exp(-\beta_j^2\bar{x}/Pe)J_0(\beta_j\bar{r})}{\beta_j^2 J_0(\beta_j)}, \\ J_1(\beta_j) &= 0. \end{aligned}\right\} \quad (18)$$

For the same case, the temperature field is approximated with only a single unknown function $g(\bar{x})$ in [1] as

$$\frac{T-T_0}{cq_w/\lambda} = 2\bar{x}/Pe+\bar{r}^{g+2}/(g+2)-2/[(g+2)(g+4)]. \quad (19a)$$

The function $g(\bar{x})$ was determined by employing the Galerkin–Kantorowich method of variational calculus as

$$(\bar{x}/Pe) = (1/8)\ln(1+4/g)-(1/18)\ln(1+3/g) -(1/2)/(g+4)+(1/6)/(g+3). \quad (19b)$$

To assess the accuracy of the present local Nusselt numbers, they are compared with the results based on equations (17) to (19) for the extreme values Bi in Table 1. It was found that the accuracy of the results depends strongly upon the number of terms considered in equations (6), (17) and (18): closer a location to the channel entrance, larger the number of terms required to achieve a good approximation for this location. From Table 1, one can conclude that the present local Nusselt numbers for $Bi < 1$ are very close to those for constant wall heat flux. For the special case of constant wall temperature ($Bi \to \infty$) and for $Bi < 1$, it can be inferred that the present local Nusselt numbers based on the first twenty terms in equation (6) agree well with the results based on the first fifty terms in equation (17) and equation (18) respectively for $(\bar{x}/Pe) > 0.0005$. This accuracy seems to be reasonable for the engineering purposes.

The Nusselt numbers based on equation (19) are also included in Table 1. They agree fairly well with the results based on equation (18).

Table 1. Local Nusselt numbers for slug flow and different values of Bi

$\bar{x}/Pe$	$Bi \to \infty$ $N = 50$ in equation (17)	∞	100	10	1	0.1	0.01	Constant wall heat flux $N = 50$ in equation (18)	[1]
		$N = 20$ in equation (6)							
0.0002	81.36	64.79	82.41	111.4	117.1	117.7	117.8	126.1	124.7
0.0005	52.07	49.73	59.75	76.60	80.42	80.84	80.88	80.91	79.75
0.001	37.32	37.12	42.76	54.58	58.01	58.41	58.45	58.55	57.09
0.002	26.87	26.87	29.89	38.42	41.67	42.07	42.11	42.20	41.09
0.005	17.67	17.66	18.95	24.20	27.17	27.58	27.62	27.71	26.94
0.01	13.07	13.07	13.75	17.20	19.91	20.33	20.37	20.42	19.87
0.02	9.883	9.882	10.24	12.44	14.85	15.28	15.32	15.36	14.96
0.05	7.237	7.236	7.393	8.544	10.54	10.99	11.04	11.29	10.83
0.1	6.179	6.177	6.264	6.956	8.626	9.099	9.154	9.165	9.054
0.2	5.817	5.816	5.871	6.319	7.690	8.173	8.231	8.234	8.216
0.5	5.783	5.783	5.834	6.230	7.458	7.937	7.996	8.022	8.003
1.0	5.783	5.783	5.834	6.230	7.456	7.935	7.993	8.002	8.000

Table 2. Local Nusselt numbers for Hagen–Poiseuille flow and different values of Bi

$\bar{x}/Pe$	$Bi \to \infty$ [5]	∞	100	10	1	0.1	0.01	Constant wall heat flux [5]	[1]
		$N = 20$ in equation (6)							
0.0002	28.25	29.51	30.14	32.78	34.40	34.62	34.64	34.51	33.79
0.0005	20.62	20.64	21.25	23.64	25.09	25.30	25.32	25.52	24.71
0.001	16.25	16.30	16.71	18.45	19.77	19.99	20.01	20.17	19.51
0.002	12.82	12.84	13.11	14.39	15.59	15.80	15.82	15.81	15.40
0.005	9.395	9.399	9.548	10.38	11.41	11.62	11.65	11.75	11.32
0.01	7.470	7.471	7.567	8.154	9.055	9.269	9.293	9.379	9.021
0.02	6.002	6.002	6.063	6.472	7.249	7.466	7.491	7.494	7.174
0.05	4.641	4.639	4.673	4.922	5.550	5.771	5.798	5.845	5.657
0.1	4.005	4.004	4.026	4.194	4.718	4.940	4.968	5.007	4.892
0.2	3.710	3.709	3.724	3.838	4.262	4.481	4.510	4.514	4.490
0.5	3.657	3.657	3.669	3.763	4.126	4.334	4.363	4.370	4.372
1.0	3.657	3.657	3.669	3.763	4.124	4.331	4.360	4.364	4.364

Table 3. Local Nusselt numbers for different values of Ha and Bi

$\bar{x}/Pe$	$Bi \to \infty$	100	10	1	0.1	0.01
			$N = 10$ in equation (6)			
			$Ha = 4$			
0.0005	24.99	26.75	30.33	31.38	31.43	31.67
0.001	20.10	20.88	22.38	23.28	23.49	23.64
0.002	15.08	15.39	16.58	17.87	17.97	18.08
0.005	10.61	10.80	11.82	12.97	13.19	13.23
0.01	8.391	8.517	9.249	10.27	10.50	10.51
0.02	6.707	6.788	7.306	8.203	8.434	8.456
0.05	5.162	5.207	5.526	6.270	6.509	6.537
0.1	4.454	4.483	4.701	5.333	5.579	5.609
0.2	4.143	4.162	4.311	4.833	5.079	5.111
0.5	4.094	4.110	4.237	4.694	4.931	4.963
1.0	4.094	4.110	4.236	4.692	4.928	4.960
			$Ha = 10$			
0.0005	28.22	30.79	37.12	38.78	38.86	38.97
0.001	23.82	25.31	28.37	29.56	29.69	29.75
0.002	18.53	19.20	20.90	22.23	22.43	22.53
0.005	12.80	13.10	14.46	15.81	15.99	16.04
0.01	9.974	10.17	11.20	12.44	12.68	12.75
0.02	7.896	8.020	8.769	9.888	10.14	10.17
0.05	6.015	6.084	6.556	7.522	7.797	7.829
0.1	5.178	5.222	5.546	6.391	6.680	6.716
0.2	4.837	4.866	5.092	5.808	6.103	6.140
0.5	4.793	4.819	5.014	5.658	5.948	5.986
1.0	4.793	4.819	5.014	5.656	5.946	5.984
			$Ha = 20$			
0.0005	30.75	33.99	43.01	45.55	45.84	46.18
0.001	26.69	28.87	33.99	35.56	35.78	36.06
0.002	21.40	22.57	25.36	26.82	27.05	27.26
0.005	14.83	15.31	17.16	18.71	18.97	19.08
0.01	11.34	11.63	13.06	14.54	14.81	14.88
0.02	8.829	9.013	10.06	11.43	11.72	11.76
0.05	6.607	6.704	7.358	8.576	8.892	8.929
0.1	5.649	5.709	6.149	7.226	7.564	7.604
0.2	5.282	5.321	5.622	6.539	6.889	6.932
0.5	5.240	5.275	5.538	6.366	6.713	6.757
1.0	5.240	5.275	5.537	6.365	6.711	6.755
			$Ha = 40$			
0.0005	32.92	36.83	48.52	52.07	52.67	54.87
0.001	28.98	31.81	39.27	41.51	41.89	43.15
0.002	23.61	25.31	29.72	31.51	31.82	32.44
0.005	16.39	17.15	19.82	21.64	21.94	22.16
0.01	12.33	12.76	14.74	16.52	16.84	16.94
0.02	9.433	9.690	11.10	12.78	13.11	13.17
0.05	6.949	7.075	7.919	9.404	9.766	9.814
0.1	5.918	5.991	6.537	7.838	8.226	8.273
0.2	5.543	5.591	5.955	7.052	7.453	7.502
0.5	5.505	5.548	5.867	6.856	7.254	7.303
1.0	5.505	5.548	5.866	6.854	7.252	7.301

Hagen–Poiseuille flow (Ha $\to$ 0)

In Table 2, the local Nusselt numbers are depicted for the Hagen–Poiseuille flow. Shah [5] calculated the local Nusselt numbers for the case of constant wall temperature ($Bi \to \infty$) and for the case of constant wall heat flux ($Bi \to 0$) by employing first 121 terms in Graetz–Nusselt type series solution. His results are also inserted in Table 2, from which one can conclude that the present results based upon first twenty terms in equation (6) agree well with the Shah's results based on 121 Eigenvalues for $(\bar{x}/Pe) > 0.0001$.

For the case of constant wall heat flux, an approximate solution similar to equation (19) is given in [1] as follows

$$
\left.
\begin{aligned}
&(T-T_0)/(cq_w/\lambda) \\
&\quad = 2\bar{x}/Pe + \frac{(g+4)\bar{r}^{g+2}}{2(g+2)} - \frac{(g+2)\bar{r}^{g+4}}{2(g+4)} \\
&\qquad - \frac{4}{(g+6)}\left[\frac{1}{(g+2)} - \frac{(g+2)}{(g+4)(g+8)}\right], \\
&\bar{x} \to 0:\ g \to \infty, \\
&\bar{x} \to \infty:\ g \to 0.
\end{aligned}
\right\} \tag{20}
$$

The function $g(\bar{x})$ was determined by employing the Galerkin–Kantorowich method of variational calculus.

From Table 2, one can conclude a satisfactory agreement between the Nusselt numbers based on equation (2) with the present results for $Bi < 1$ and with the results reported by Shah [5].

Hartmann flow ($0 < \mathrm{Ha} < \infty$)

In Table 3, the local Nusselt numbers are presented for a few arbitrarily selected values of Ha and Bi. Here, only first ten terms in a series solution (6) were selected to save calculation time, since no substantial difference between the results based on first twenty terms and the results based on first ten terms in equation (16) for $(\bar{x}/Pe) > 0.0005$ was observed. The results given in Table 3 cannot be compared, as no similar analysis is, to the author's knowledge, reported in literature. Based on the conclusion from Tables 1 and 2, the local Nusselt numbers for $Bi < 1$ are expected to be very close to those for the constant wall heat flux.

Finally, from the results obtained, one can infer, that there can be a significant influence of the finite wall thermal resistance on the magnetohydrodynamic heat transfer in a circular duct and the analysis of the problem by neglecting this effect may result in considerable error in the solutions representing the actual physical situations.

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APPENDIX

The elements of the vectors and matrices occuring in equations (9) and (11) are listed below.

$i = 1, 2, \ldots N.$

$j = 1, 2, \ldots N.$

$a = s_i, \quad b = s_j, \quad c = a^2 - b^2,$

$y_0 = J_0(a), \quad y_1 = J_1(a), \quad y_2 = J_2(a),$

$z_0 = J_0(b), \quad z_1 = J_1(b), \quad z_2 = J_2(b).$

$$t_1 = \int_0^1 J_0(a\bar{r})\bar{r}\,\mathrm{d}\bar{r} = y_1/a.$$

$$t_3 = \int_0^1 J_0(a\bar{r})\bar{r}^3\,\mathrm{d}\bar{r} = (-4y_1 + a^2 y_1 + 2ay_0)/a^3.$$

$$\begin{aligned} q_1 &= \int_0^1 J_0(a\bar{r})J_0(b\bar{r})\bar{r}\,\mathrm{d}\bar{r} \\ &= (y_0^2 + y_1^2)/2, && \text{if } a = b; \\ &= (ay_1z_0 - by_0z_1)/c, && \text{if } a \neq b. \end{aligned}$$

$$\begin{aligned} p_1 &= \int_0^1 J_0(a\bar{r})J_1(b\bar{r})\bar{r}^2\,\mathrm{d}\bar{r} \\ &= (y_0 + y_2)y_1/4, && \text{if } a = b; \\ &= (ay_1z_1 + by_0z_0)/c - 2bq_1/c, && \text{if } a \neq b. \end{aligned}$$

$$\begin{aligned} p_2 &= \int_0^1 J_1(a\bar{r})J_0(b\bar{r})\bar{r}^2\,\mathrm{d}\bar{r} \\ &= (y_0 + y_2)y_1/4, && \text{if } a = b; \\ &= -(by_1z_1 + ay_0z_0)/c + 2aq_1/c, && \text{if } a \neq b. \end{aligned}$$

$$\begin{aligned} q_3 &= \int_0^1 J_0(a\bar{r})J_0(b\bar{r})\bar{r}^3\,\mathrm{d}\bar{r} \\ &= (y_0^2 + y_1^2)/4 - (y_1^2 + y_2^2)/12, && \text{if } a = b; \\ &= (ay_1z_0 - by_0z_1)/c + 2bp_1/c - 2ap_2/c, && \text{if } a \neq b. \end{aligned}$$

$W(i,j) = -b^2 q_1.$

$\mathrm{Ha} \to \infty$

$R(i) = t_1, \quad D(i,j) = q_1.$

$\mathrm{Ha} \to 0$

$R(i) = 2(t_1 - t_3), \quad D(i,j) = 2(q_1 - q_3).$

$0 < \mathrm{Ha} < \infty$

$h_0 = I_0(Ha), \quad h_1 = I_1(Ha), \quad H_0 = Ha/(Hah_0 - 2h_1).$

$$\begin{aligned} t_5 &= \int_0^1 I_0(Ha\bar{r})J_0(a\bar{r})\bar{r}\,\mathrm{d}\bar{r} \\ &= (Hah_1y_0 + ah_0y_1)/[(Ha)^2 + a^2] \end{aligned}$$

$$q_7 = \int_0^1 I_0(Ha\bar{r})J_0(a\bar{r})J_0(b\bar{r})\bar{r}\,\mathrm{d}\bar{r}.$$

$R(i) = H_0(h_1t_1 - t_5), \quad D(i,j) = H_0(h_0q_1 - q_7).$

TRANSFERT THERMIQUE MAGNETOHYDRODYNAMIQUE DANS UN TUBE CIRCULAIRE AVEC UNE CONDITION A LA LIMITE DU TROISIEME ORDRE POUR LA TEMPERATURE

Résumé—Pour étudier l'influence de la condition du troisième ordre, pour la température, sur le transfert thermique magnétohydrodynamique à l'entrée d'un tube circulaire, on résout l'équation d'énergie en appliquant la méthode de Galerkin–Kantorowich du calcul variationnel. On suppose le profil de vitesse pleinement développé et on néglige la génération de chaleur dans le fluide. On conclut qu'il peut y avoir une influence sensible du nombre de Biot sur le nombre de Nusselt. Des résultats représentatifs sont donnés dans des tables.

MAGNETOHYDRODYNAMISCHE WÄRMEÜBERTRAGUNG IN EINEM KREISROHR FÜR TEMPERATUR-RANDBEDINGUNGEN DRITTER ART

Zusammenfassung—Um den Einfluß der Temperatur-Randbedingung dritter Art auf die magnetohydrodynamische Wärmeübertragung im thermischen Einlaufgebiet eines Kreisrohrs zu untersuchen, wird die Energiegleichung mit dem Kantorowitsch-Galerkin-Verfahren der Variationsrechnung gelöst. Die ausgebildete Geschwindigkeitsverteilung wird angenommen und die Wärmeentwicklung im Fluid wird vernachlässigt. Es wurde festgestellt, daß es einen wesentlichen Einfluß der Biot-Zahl auf die lokale Nusselt-Zahl geben kann. Typische Ergebnisse sind in den Tabellen dargestellt.

МАГНИТОГИДРОДИНАМИЧЕСКИЙ ТЕПЛООБМЕН В КРУГЛОМ КАНАЛЕ ПРИ ТЕПЛОВОМ ГРАНИЧНОМ УСЛОВИИ ТРЕТЬЕГО РОДА

Аннотация — Для исследования влияния теплового граничного условия третьего рода на магнитогидродинамический теплообмен на начальном тепловом участке круглого канала решается уравнение энергии вариационным методом Галеркина-Канторовича. Предполагается, что профиль скорости является полностью развитым. Генерацией тепла в жидкости пренебрегается. Найдено, что число Био может оказывать значительное влияние на локальное число Нуссельта. Характерные результаты представлены в таблицах.